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ROBUST MULTI-OBJECTIVE FILTERING FOR A CLASS OF NEUTRAL SYSTEM WITH BOUNDED NOISE AND PARAMETRIC UNCERTAINTY

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Abstract:

This paper considers a robust H_∞ filtering problem for a class of neutral delay differential uncertain systems with polytope type uncertainty. The main purpose is to obtain a stable linear filter such that the filtering error system remains robustly stable within a pre-specific H_∞ attenuation level or minimum H_∞ attenuation level γ. Sufficient conditions are stated in terms of LMIs. Based on the LMI toolbox, the solutions of the inequalities are obtained. Moreover, as an improvement, the filtering error system dynamics can be constrained to some specific LMI regions. The validity of proposed filter design algorithm has been checked through a numerical example.

Keywords:

Uncertainty; Neutral delay system; Filtering; Linear matrix inequality; Pole constraint

1. Introduction

The problem of state estimation and filtering for system has been one of the fundamental issues in the control area. There has been a lot of interest on the problem of robust H_∞ filtering for dynamics systems subject to unknown but bounded noise and parameters uncertainty affecting possibly every system matrix. Several approaches exist in the literature up to this data, e.g., the stochastic approach (Kalman filtering theory), H_∞ filtering theory and deterministic, or set-membership, approach. In the extensively studied H_∞ filtering, the exogenous input signal is assumed to be energy bounded rather than Gaussian, and the filter is designed such that the worst case "gain" of the system is minimized [1,2]. On the other hand, delay equation/systems arising a variety of fields , such as biology electro-dynamics, have received considerable attention. The theory of neutral delay differential systems is both theoretical and practical interest [3,4]. The stability analysis for neutral delay differential of various types has been extensively studied [5,6]. Various methods have been introduced to derive less conservative and concise stability criteria for neutral system [7,8]. The H_∞ filtering problem for a class of neutral systems has been also investigated in Park papers [8,9]. But it has not been fully investigated and remains to be important and challenging.

In the literature, two kinds of uncertainties in the parameters have been studied, i.e., the norm bounded uncertainties and the convex polytopic ones. Subject to the latter case, the problem of robust H_∞ filtering for neutral delay differential systems is investigated in this paper. Moreover, as an improvement, robust pole placement specifications is considered, that ensures fast and well-damped transient response etc. By using Lyapunov functional technique combined with linear matrix inequalities technique, the problems to be addressed are the robust H_∞ filtering problem (P_∞) and the robust H_∞ filtering with pole placement constraint (P_∞^R).

- P_∞ designs a stable linear filter ensuring the error system asymptotically stable and finding a minimized or pre-specified positive attenuation level γ,

$$\sup_{\omega\in l_2[0,\infty)} \frac{\|z_e(t)\|_2}{\|\omega(t)\|_2} < \gamma \qquad (1)$$

with poly type parameters uncertainties.

- P_∞^R determine a stable linear filter ensuring (1) and all eigenvalues of matrix A_e are inside some particular region of the complex plane.

2. Problem formulation

Consider a class of neutral differential system of the form:

$$\dot{x}(t) = A_0x(t) + A_1x(t-h) + A_2\int_{t-\tau}^{t} x(s)ds + A_3\dot{x}(t-\varepsilon) + Bw(t)$$
$$y(t) = C_yx(t) + D_yw(t)$$
$$z(t) = C_zx(t)$$
$$x(t_0+\theta) = \phi(\theta) \quad \forall\theta \in [-H,0] \tag{2}$$

where $x(t) \in R^n$ is the state vector, $w(t) \in R^m$ is the disturbance signal in L_2, $y(t) \in R^l$ is the measured output, $z(t) \in R^q$ is the estimated state vectors, h, τ and ε are the positive constant time delays, $H = \max\{h,\tau,\varepsilon\}$, $\phi(\theta) \in L_0$ is the initial vector, where L_0 is a set of all continuous differentiable function on $[-H,0]$ to R^n. The system matrices A_0, A_1, A_2, A_3, B, C_y, D_y, C_z are assumed to be unknown (uncertain) but belonging to a known convex compact set of polytopic type, i.e.

$$D = \{(A_0, A_1, A_2, A_3, B, C_y, D_y, C_z) = \sum_{i=1}^{l}\tau_i(A_{0i}, A_{1i}, A_{2i}, A_{3i}, B_i, C_{yi}, D_{yi}, C_{zi}); \tau_i \ge 0; \sum_{i=1}^{l}\tau_i = 1\} \tag{3}$$

This kind of convex bounded parameter uncertainties has been fairly investigated in [2]. In this paper, the aim is to design a robust H_∞ filtering to estimate the unmeasured signal $z(t)$ and the H_∞ norm of filtering error system is minimized or less than a prescribed γ attenuation level. For that, a stable linear filter describes:

$$\dot{\hat{x}}(t) = A_f\hat{x}(t) + A_1\hat{x}(t-h) + A_2\int_{t-\tau}^{t}\hat{x}(s)ds + A_3\dot{\hat{x}}(t-\varepsilon) + B_fy(t)$$
$$\hat{x}(0) = 0, \quad s \in [-H,0] \tag{4}$$

where $\hat{x}(t) \in R^n$ denotes the state estimation and A_f, B_f are the filter parameters to be determined. The state of estimation error define as $\bar{x}(t) = x(t) - \hat{x}(t)$. Refine the augmented state vector as

$$x_e(t) = \begin{bmatrix} x(t) \\ \bar{x}(t) \end{bmatrix},$$

and the estimation error as

$$z_e(t) = z(t) - \hat{z}(t),$$

such that the filtering error dynamics can be written as the following:

$$\dot{x}_e(t) = A_ex_e(t) + A_{1e}x_e(t-h) + A_{2e}\int_{t-\tau}^{t}x_e(s)ds + A_{3e}\dot{x}_e(t-\varepsilon) + B_ew(t) \tag{5}$$
$$z_e(t) = C_ex_e(t) + D_ew(t) \tag{6}$$

where
$$A_e = \begin{bmatrix} A_0 & 0 \\ A_0 - B_fC_y - A_f & A_f \end{bmatrix},$$
$$A_{1e} = \begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix}, A_{2e} = \begin{bmatrix} A_2 & 0 \\ 0 & A_2 \end{bmatrix}, A_{3e} = \begin{bmatrix} A_3 & 0 \\ 0 & A_3 \end{bmatrix},$$
$$B_e = \begin{bmatrix} B \\ B - B_fD_y \end{bmatrix}, \qquad C_e = [0 \quad C_z].$$

Here, the H_∞ performance is defined as :

$$J = \int_0^\infty [z_e^T(t)z(t) - \gamma^2w^T(t)w(t)]dt \tag{7}$$

3. Robust H_∞ filtering

In this section using the Lyapunov stability theory, we present the robust H_∞ filter design method of neutral differential systems with uncertainties, which belong to the polytopic uncertainty. Before designing the H_∞ filter, we introduce the useful lemmas.

Lemma 1 (D. Yue, S. Won and O. Kwon [3]) For positive scalar h, τ and ε and $E_1, E_2, E_3 \in R^{n\times n}$, the operator $D(x_t): L_0 \to R^n$ defined by

$$D(x_t) = x(t) + E_1\int_{t-h}^{t}x(s)ds + E_2\int_{t-\tau}^{t}(s-t-\tau)x(s)ds - E_3x(t-\varepsilon) \tag{8}$$

The operator $D(x_t): L_0 \to R^n$ is stable if there exist a positive definite matrix Γ and positive scalars α_1, α_2 and α_3 such that

$$\begin{bmatrix} E_3^T\Gamma E_3 - a_1\Gamma & hE_3^T\Gamma E_1 & \tau E_3^T\Gamma E_2 \\ * & h^2E_1^T\Gamma E_1 - a_2\Gamma & \tau E_1^T\Gamma E_2 \\ * & * & \tau^2E_2^T\Gamma E_2 - \frac{3\alpha_3\Gamma}{\tau^2} \end{bmatrix} < 0$$
$$\alpha_1 + \alpha_2 + \alpha_3 < 1 \tag{9}$$

Lemma 2: (J.Park [5]) For any constant symmetric positive-definite matrix Ψ, a scalar σ, and the vector function $\varphi[0, \ \sigma] \to R^m$ such that the integrations in the following are well defined, then

$$\sigma\int_0^\sigma \varphi^T(s)\Psi\varphi(s) \ge \left(\int_0^\sigma \varphi(s)ds\right)^T \psi\left(\int_0^\sigma \varphi(s)ds\right) \tag{10}$$

Thus, the filtering error system (5) can be rewritten in the following form:

$$\begin{aligned}&\frac{d}{dt}(x_e(t)+A_{1e}\int_{t-h}^{t}x_e(s)ds-A_{3e}x_e(t-\varepsilon)\\&+A_{2e}\int_{t-\tau}^{t}(s-t+\tau)x_e(s)ds)=Ax_e(t)+B_e w(t).\end{aligned} \tag{11}$$

With $A=A_e+A_{1e}+\tau A_{2e}$.

Define the operator $D(x_{et}):l_0\rightarrow R^n$ as

$$\begin{aligned}D(x_{et})=x_e(t)+A_{1e}\int_{t-h}^{t}x_e(s)ds-A_{3e}x_e(t-\varepsilon)\\+A_{2e}\int_{t-\tau}^{t}(s-t+\tau)x_e(s)ds\end{aligned} \tag{12}$$

In Park work[5], a robust H_∞ filter for nominal neutral delay differential system is developed that makes the filtering error system asymptotically stable and the gives the up bound H_∞ norm of the T_{we}. In that reference, the input signal is assumed to be arbitrary deterministic signal of bounded energy and the parameters of neutral system are precisely known. This means the system is nominal. Similarly to the Theorem 2 of the Park [5], a robust H_∞ performance and one with pole placement constraint for neutral systems with parameter uncertainties are developed in this paper.

In the following Theorem, the asymptotic stability and a criterion for the existence of a robust linear stable filter assuring a robust H_∞ performance are established.

Based on the Lemma 1, the operator $D(x_{et})$ of the filtering error system (5) is stable if there exists a positive definite matrix H and scalars a_1 and a_2 such that

$$\begin{bmatrix}A_{3e}^T\Gamma A_{3e}-a_1\Gamma & hA_{3e}^T\Gamma A_{1e} & \tau A_{3e}^T\Gamma A_{2e}\\ * & h^2A_{1e}^T\Gamma A_{1e}-a_2\Gamma & \tau A_{1e}^T\Gamma A_{2e}\\ * & * & \tau^2A_{2e}^T\Gamma A_{2e}-\dfrac{3\alpha_3\Gamma}{\tau^2}\end{bmatrix}<0 \tag{13a}$$

$$\alpha_1+\alpha_2+\alpha_3<0 \tag{13b}$$

$$\forall(A_0,A_1,A_2,A_3,B,C_y,D_y,C_z)\in D$$

According to the Lyapunov stable theory, the asymptotic stability of the filtering error system, for all $\forall(A_0,A_1,A_2,A_3,B,C_y,D_y,C_z)\in D$ is established in the following Theorem.

Theorem 1: For the given neutral delay uncertainty system (3) with delay time scalars $h>0,\tau>0$ and $\varepsilon>0$. Suppose that the operator $D(x_{et})$ is stable. If there exist the positive definite $P_1,P_2,Y_1,Y_3,S_1,S_3,H_1,H_3,\gamma$ and mtrices $\mathrm{M}_1,\mathrm{M}_2,S_2,Y_2,H_2$ satisfying the following LMIs:

$$\begin{bmatrix}\Omega_1 & \Pi_1 & \tau A_i^TP_1A_{2i} & \Omega_2A_{2i} & -A_i^TP_1A_{3i} & -\Omega_2A_{3i} & hA_i^TP_1A_{1i} & h\Omega_2A_{1i} & P_1B_i & 0 & 0\\ * & \Pi_2 & 0 & \Omega_3A_{2i} & 0 & -\Omega_3A_{3i} & 0 & h\Omega_3A_{1i} & P_2B_i-M_2D_{yi} & 0 & 0\\ * & * & H_1 & -H_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -H_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -S_1 & -S_2 & 0 & 0 & -A_{3i}^TP_1B_i & 0 & 0\\ * & * & * & * & * & -S_3 & 0 & 0 & \Pi_4 & 0 & 0\\ * & * & * & * & * & * & -Y_1 & -Y_2 & hA_{1i}^TP_1B_i & 0 & 0\\ * & * & * & * & * & * & * & -Y_3 & \Pi_3 & 0 & 0\\ * & * & * & * & * & * & * & * & -\gamma I & \tau B_i^TP_1A_{2i} & \Omega_4\\ * & * & * & * & * & * & * & * & * & -H_1 & -H_2\\ * & * & * & * & * & * & * & * & * & * & -H_3\end{bmatrix}<0 \tag{14a}$$

$$\begin{bmatrix}Y_1 & Y_2\\ Y_2^T & Y_3\end{bmatrix}>0,\quad\begin{bmatrix}S_1 & S_2\\ S_2^T & S_3\end{bmatrix}>0,\quad\begin{bmatrix}H_1 & H_2\\ H_2^T & H_3\end{bmatrix}>0 \tag{14b}$$

$$\forall(A_0,A_1,A_2,A_3,B,C_y,D_y,C_z)\in D$$

then the filtering error system (5,6) is asymptotically stable and satisfies the robust H_∞ filtering problem P_∞ when the parameters of system(3) belong to the poly type uncertainties. The filter is a robust H_∞ filter and determined by:

$$A_f = P_2^{-1} M_1 \tag{15a}$$

$$B_f = P_2^{-1} M_2 \tag{15b}$$

Where $A_i = A_{0i} + A_{1i} + \tau A_{2i}$

$$\Omega_1 = P_1 A_i + A_i^T P_1 + Y_1 + S_1 + \frac{1}{2}\tau^2 H_1$$

$$\Omega_2 = A_{0i}^T P_2 - C_{yi}^T M_2^T - M_1^T$$

$$\Omega_3 = A_{1i}^T P_2 + \tau A_{2i}^T P_2 + M_1^T$$

$$\Omega_4 = \tau B_i^T P_2 A_{2i} - \tau D_{yi}^T M_2^T A_{2i}$$

$$\Pi_1 = \Omega_2 + Y_2 + S_2 - \tau^2 H_2$$

$$\Pi_2 = \Omega_3 + \Omega_3^T + Y_3 + S_3 + \tau^2 H_3 + C_{zi}^T + C_{zi}$$

$$\Pi_3 = hA_{1i}^T P_2 B_i - hA_{1i}^T M_2 D_{yi}$$

$$\Pi_4 = -A_{3i}^T P_2 B_i + A_{3i}^T M_2 D_{yi}$$

Proof : Consider the following legitimate Lyapunov function:

$$V(t, x_e(t)) = D^T(x_{et}) P D(x_{et}) + \int_{t-h}^{t} (s-t+h) x_e^T(s) Y x_e(s) d(s) + \frac{1}{2}\int_{t-\tau}^{t} (s-t+\tau) x_e^T(s) H x_e(s) d(s) + \int_{t-\varepsilon}^{t} x_e^T(s) S x_e(s) d(s) \tag{16}$$

where $P > 0,\ Y > 0, S > 0, H > 0$. Obviously $V(t, x_e(t)$ is positive definite. Based on the Lyapunov stability theorem, if $\frac{dy}{dt}(t, x_e(t))$ is semi-negative for $\forall t > 0$, then the system is asymptotically stable. Based on the operator (12),

$$\frac{dV}{dt}(t, x_e(t)) = 2x_e^T(t) A^T P\{x_e(t) + A_{1e}\int_{t-h}^{t} x_e(s)ds - A_{3e} x_e(t-\varepsilon) + A_{2e}\int_{t-\tau}^{t}(s-t+\tau)x_e(s)ds\} - \int_{t-h}^{t} x_e^T(s) Y x_e(s) ds - x_e^T(t-\varepsilon) S x_e(t-\varepsilon) + \frac{1}{2}\tau^2 x_e^T(t) H x_e(t) + h x_e^T(t) Y x_e(t) + x_e^T(t) S x_e(t) - \int_{t-\tau}^{t} x_e^T(s) H x_e(s) ds \tag{17}$$

Using Lemma 2, the following form is obtained

$$\int_{t-h}^{t} x_e^T(s) Y x_e(s) ds \geq \left(\frac{1}{h}\int_{t-h}^{t} x_e(s)ds\right)^T (hY) \left(\frac{1}{h}\int_{t-h}^{t} x_e(s)ds\right) \tag{18}$$

And

$$2x_e^T(t) A^T P A_{2e}\int_{t-\tau}^{t}(s-t+\tau)x_e(s)ds \leq \int_{t-\tau}^{t}(s-t+\tau)[2x_e^T(s) A^T P A_{2e}^T H^{-1} A_{2e} P A x_e(s) + \frac{1}{2} x_e^T(s) H x_e(s)]ds \leq \tau^2 x_e^T(t) A^T P A_{2e}^T H^{-1} A_{2e} P A x_e(t) + \frac{1}{2}\int_{t-\tau}^{t}(s-t+\tau) x_e^T H x_e(s) ds \tag{19}$$

Substituting (18) and (19) into $\frac{dV}{dt}(t, x_e(t)$, the following is hold.

$$\frac{dV}{dt}(t, x_e(t)) \leq x_e^T(t)(PA + A^T P + hY + S + \frac{1}{2}\tau^2 H + \tau^2 A^T P A_{2e}^T H^{-1} A_{2e} P A) x_e(t) + 2x_e^T(t) A^T P A_{1e}\int_{t-h}^{t} x_e(s)ds - 2x_e^T(t) A^T P A_{3e} x_e(t-\varepsilon) - x_e^T(t-\varepsilon) S x_e(t-\varepsilon) - \left(\frac{1}{h}\int_{t-h}^{t} x_e(s)ds\right)^T (hY)\left(\frac{1}{h}\int_{t-h}^{t} x_e(s)ds\right) - \frac{1}{2}\int_{t-\tau}^{t}(s-t+\tau)x_e^T H x_e(s)ds \tag{20}$$

If $\frac{dV}{dt} \leq 0$, the filtering error system(6) is stable.

The next we give the H_∞ performance for error system (6). When the filtering error system is stable, the states of error system $x_e(t)$ tend to zero as $t \to \infty$. Now assuming zero initial conditions for the filtering error system, the H_∞ performance index is defined as following:

$$J = \int_0^\infty z_e^T(t) z_e(t) - \gamma^2 w^T(t) w(t) + \dot{V}(x_e(t), t) dt .$$

$\forall w(t) \in L_2[0, \infty]$, which is bounded energy stochastic signal. When $J < 0$, the L_2 gain of filtering error system (6) is smaller than γ. Based on the legitimate Lyapunov functional candidate (16), we can easily infer the H_∞

performance index *J:*

$$J \le \int_0^\infty (x_e^T(t)C_e^T C_e x_e + x_e^T(t)(PA + A^T P + hY + S + \frac{1}{2}\tau^2 H + \tau^2 A^T PA_{2e}^T H^{-1} A_{2e} PA)x_e(t) + 2x_e^T(t)A^T PA_{1e}\int_{t-h}^{t} x_e(s)ds - 2x_e^T(t)A^T PA_{3e}x_e(t-\varepsilon) - x_e^T(t-\varepsilon)Sx_e(t-\varepsilon) - (\frac{1}{h}\int_{t-h}^{t} x_e(s)ds)^T (hY)(\frac{1}{h}\int_{t-h}^{t} x_e(s)ds) - \frac{1}{2}\int_{t-\tau}^{t}(s-t+\tau)x_e^T Hx_e(s)ds + 2w^T(t)(-\gamma^2 + \tau^2 B_e^T PA_{2e}H^{-1}A_{2e}PB_e)w(t))dt \quad (21)$$

Then the objective function equivalent to:

$$\int_0^\infty \begin{bmatrix} x_e(t) \\ x_e(t-\tau) \\ \frac{1}{h}A_1\int_{t-h}^{t} x_e(s)ds \\ w(t) \end{bmatrix}^T \mathrm{M} \begin{bmatrix} x_e(t) \\ x_e(t-\tau) \\ \frac{1}{h}A_1\int_{t-h}^{t} x_e(s)ds \\ w(t) \end{bmatrix} dt \le 0 \quad (22)$$

Where the M is:

$$\begin{bmatrix} \phi_1 & * & -A_{3e}^T PB_e & * \\ -A_{3e}^T PA & -S & * & * \\ hA_{1e}^T PA & 0 & -hY & * \\ B_e^T P & * & hB_e^T PA_{1e} & -\gamma^2 I + \tau^2 B_e^T PA_{2e}H^{-1}A_{2e}PB_e \end{bmatrix} \quad (23)$$

$$\phi_1 = A^T P + PA + hY + S + C_e^T C_e + \frac{1}{2}\tau^2 H + \tau^2 A^T PA_{2e}^T H^{-1}A_{2e}PA$$

$J < 0$ implies to M < 0. According to Shur completement and the parameters of the filtering error system, we define the

$$Y = \begin{bmatrix} Y_1 & Y_2^T \\ Y_2 & Y_3 \end{bmatrix} > 0, \qquad S = \begin{bmatrix} S_1 & S_2^T \\ S_2 & S_3 \end{bmatrix} > 0,$$

$$H = \begin{bmatrix} H_1 & H_2^T \\ H_2 & H_3 \end{bmatrix} > 0, \qquad P = \begin{bmatrix} P_1 & 0 \\ 0 & p_2 \end{bmatrix} > 0.$$

Substituting Y, S, H, P and A, A_{1e}, A_{2e}, A_{3e}, B_e, C_e, D_e into (23) and making $M_1 = P_2 B_f$, $M_2 = P_2 A_f$, applying Shur completement again, the LMI (14) for system with uncertainty (3) is obtained.

Remark 1: It is obviously that the LMI (14) is independent of differential delay constant ε. The result in this paper can be easily extended to time-varying delays systems.

4. Pole constraint

In some case, the resulting filtering design does not present a suitable dynamical behavior. In this paper, the transient behaviors of filtering error system (5) can be improved by constraining the eigenvalues of the A_e to lie inside a given region (to ensure fast and well-damped transient responses. Because of the uncertainty existing in the error system, the pole placement in LMI regions instead of the standard pole placement, which places the poles in current dots. Define the region $L(v, \lambda)$, which can be disk, conic, strip. Consider $\forall (A_0, A_1, A_2, B, C_y, D_y, C_z) \in D$, then there exists $P = P^T$, such that

$$[\lambda_{i,j} P + v_{i,j} A_e^T P + v_{j,i} PA_e]_{1<i,j<m} < 0 \quad (24)$$

The solvability of problem P_∞^R is dressed through the next theorem:

Theorem2: Consider a class of neutral differential system with polytopic uncertainties (3), let $\gamma > 0$ be given. If there exist the positive definite matrices P_1, P_2, Y_1, Y_3, S_1, S_3, H_1, H_3 and matrices $\mathrm{M}_1, \mathrm{M}_2, S_2, Y_2, H_2$ such that the inequalities (14a,14b) and

$$\begin{bmatrix} \lambda_{i,j} P_1 + v_{i,j} A_0^T P_1 + v_{j,i} P_1 A_0 & * \\ v_{j,i} P_2 A_0 - v_{j,i} M_1 C_y - v_{j,i} M_2 & \lambda_{i,j} P_2 + v_{i,j} M_2^T + v_{j,i} M_2 \end{bmatrix} < 0 \quad (25)$$

$$\forall (A_0, A_1, A_2, A_3, B, C_y, D_y, C_z) \in D$$

are hold, then the error system satisfies the bound $\|z_e\|_2 < \gamma \|w\|_2$ and the eigenvalues of the A_f are inside the region $L(v, \lambda)$ for $\forall (A_0, A_1, A_2, A_3, B, C_y, D_y, C_z) \in D$.

The proof follows the steps of the Theorem1 and omits.

5. Example

Consider the neutral delay uncertainty system (1) with

$$A_0 = \begin{bmatrix} 0 & 3 \\ -4 & j \end{bmatrix}, A_1 = \begin{bmatrix} -0.5 & 0 \\ 0.2 & -0.5 \end{bmatrix}, A_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.1 & 0.05 \\ 0.05 & 0.1 \end{bmatrix}, B = \begin{bmatrix} -0.5 \\ 1 \end{bmatrix}, \ C_y = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$C_z = \begin{bmatrix} 0 & 1 \end{bmatrix}, D_y = [1], h_i = \alpha, \tau = 0.7, \varepsilon = 1$$

assuring that $j \in [-5, -7]$, $\alpha \in [0.3, 0.4]$. Based on the LMI toolbox, the optimal robust H_∞ filter cost achieved is $\gamma = 2.9153$ with the solution of the inequalities of (14). The parameters of the robust H_∞ filter are

$$A_f = \begin{bmatrix} -207.23 & 14.076 \\ -12.595 & -5.378 \end{bmatrix}, \qquad B_f = \begin{bmatrix} 170.213 \\ 6.7205 \end{bmatrix},$$

The eigenvalues of the robust filter in P_∞ are given by

$$\lambda(A_f) = \{-206.3474, -6.26.1\}.$$

Particularly dynamical behavior for filter can be obtained by imposing the poles of (A_f) to be constrained inside the some region. For example, the poles of A_f are constrained inside the $s > -10$. According toTheorem2 the optimal solution is $\gamma = 121.7225$ with the filter matrices

$$A_{f1} = \begin{bmatrix} -5.514 & 1.9053 \\ -1.9191 & -8.5274 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} 3.7491 \\ -3.1304 \end{bmatrix}.$$

With

$$\lambda(A_{f1}) = [-7.0209 + 1.1776i \quad -7.0209 - 1.1776i].$$

The smallest robust H_∞ guaranteed cost with pole constraint (pole place in the $s > -10$ region) is obviously bigger than one without pole constraint. It means the conservatism is increased by adding the pole constraint. Next consider that the poles are constrained inside an other LMI region $s > -20$. The optimal solution is $\gamma = 2.9557$ with the filter matrices

$$A_{f2} = \begin{bmatrix} -11.2632 & 3.6982 \\ -1.9362 & -6.0193 \end{bmatrix}, B_{f2} = \begin{bmatrix} 9.1990 \\ -2.0140 \end{bmatrix}.$$

and the eigenvalues are:

$$\lambda(A_{f2}) = \{ \text{-8.6413 + 0.5345i, -8.6413 - 0.5345i} \}.$$

It is interesting to note that the conservatism is compensated by fitting readjusting the pole placement region.

6. Conclusion

The robust H_∞ filtering for a class of neutral delay-differential uncertain systems with polytopic type uncertainty has been addressed in this paper. An LMI-based technique for designing a robust filter assuring a guaranteed H_∞ performance has been proposed. Furthermore, additional pole constraints can be imposed to filtering error system dynamics in order to improve the dynamics performance of the error system. The half plane region is considered in numerical example. The results show that the approach proposed in this paper is valid.

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